doi: 10.67563/ASESCONF2026099V

Ivana Veličković[1], Milivoje Rogač[2]

NONLINEAR ANALYSIS OF (UN)BRACED STEEL FRAMES SUBJECTED TO HORIZONTAL LOAD

Summary: This paper presents a 2D nonlinear analysis of the load-carrying capacity of unbraced and concentrically braced steel frames subjected to lateral loading. The study was conducted using the incremental ("step-by-step") analytical method and finite element analysis. The effects of material and geometric nonlinearities, axial force, and brace cross-sectional area on the structural response were investigated. Axial force had no significant influence on the formation of plastic hinges in the analyzed systems. The results show that including geometric nonlinearity and a strain-hardening material model does not significantly affect the overall structural response. The numerical models were validated against analytical solutions, with differences below 5%. Increasing the brace cross-sectional area improves both the load-carrying capacity and stiffness, without affecting the location or sequence of plastic hinge formation.

Keywords: steel frames, nonlinear analysis, bracing, plastic hinges, N–M interaction

NELINEARNA ANALIZA (NE)UKRUĆENIH ČELIČNIH RAMOVA IZLOŽENIH DEJSTVU HORIZONTALNE SILE

Rezime: U radu je prikazana nelinearna 2D analiza nosivosti neukrućenih i koncentrično ukrućenih čeličnih ramova izloženih dejstvu horizontalnog opterećenja. Istraživanje je sprovedeno primjenom analitičke metode "korak po korak" i numeričke analize metodom konačnih elemenata. Analiziran je uticaj materijalne i geometrijske nelinearnosti, normalne sile i površine poprečnog presjeka zatege na odgovor sistema. Normalna sila nema značajan uticaj na formiranje plastičnih zglobova kod analiziranih sistema. Rezultati pokazuju da uvođenje geometrijske nelinearnost i materijalnog modela sa ojačanjem ne utiče značajno na ukupni odgovor razmatranih sistema. Numerički modeli su validirani poređenjem sa analitičkim rješenjima, pri čemu su razlike bile manje od 5%. Sa povećanjem površine zatege, raste nosivost i krutost sistema. Promjena površine zatege ne utiče na položaj i redosljed formiranja plastičnih zglobova kod razmatranih sistema.

Ključne reči: čelični ramovi, nelinearna analiza, ukrućenje, plastični zglobovi, N-M interakcija

[1] Faculty of Civil Engineering, University of Montenegro, e-mail: ivanaavelickovic@gmail.com
[2] Faculty of Civil Engineering, University of Montenegro

1. INTRODUCTION

The research comprises a nonlinear analysis of the load-carrying capacity of braced and unbraced steel frames subjected to a horizontal force (Figure 1) [1]. The design of structures in seismically active regions goes beyond traditional methods of calculation, which is why nonlinearity is introduced into the analysis [2-4]. An unbraced steel frame is characterised by high ductility and low stiffness, while energy is dissipated through cyclic bending. In concentrically braced steel frames, the dissipation zones are usually located in the tension diagonals [5]. The results of previous research [6–9] indicate that braced frames achieve greater lateral load-carrying capacity and stiffness, thereby providing a more favourable response of the structure to lateral actions.

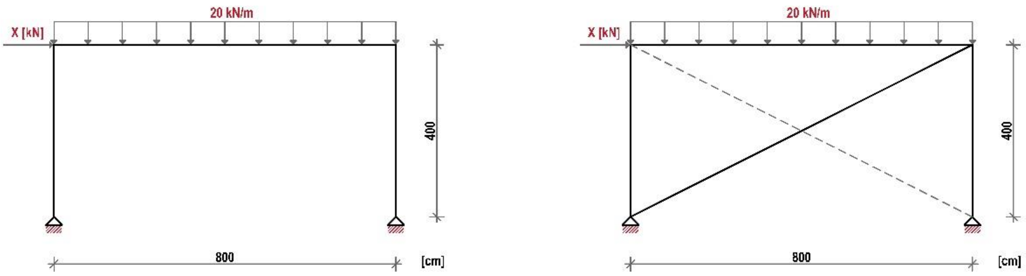


Figure 1: Unbraced (left) and braced (right) steel frame

The analysis relies predominantly on analytical and numerical solutions (*Abaqus*) [10]. Within the analytical and numerical model that was formed, a fixed frame beam span of 8 m and a frame column height of 4 m were analysed (Figure 1). The cross-section of the beam and of the column is identical and constant, a welded I-cross-section (Figure 2). The vertical load is constant, uniformly distributed along the frame beam (20 kN/m), while the horizontal load (acting at one beam-column joint) increases until the loss of load-carrying capacity. The adopted load arrangement approximately simulates, in a static manner, the dynamic seismic loading of frames in service. The analysis covers an unbraced and a concentrically braced steel frame. The bracing configuration is fixed and corresponds to X bracing, whereby one member is excluded from the analysis (the compressed member for the defined direction of the horizontal force – the dashed line in Figure 1- right). The area of the tie rod in the analytical and numerical calculation is A = 3.89 cm². As presented in Figure 1, both systems are two-hinged frames with following degrees of freedom at the column bases: restrained horizontal and vertical displacements and free rotations.

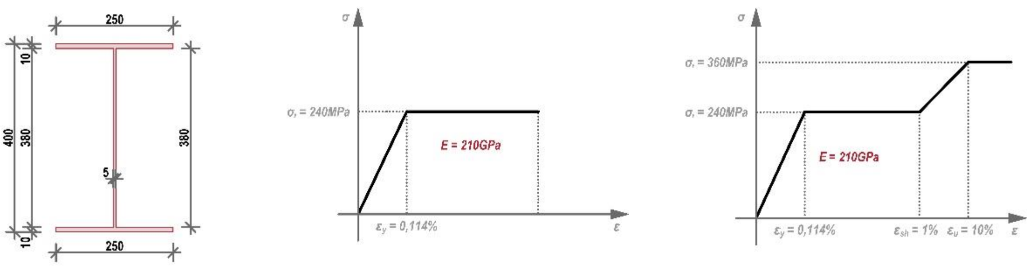


Figure 2: Cross-section and material models

The material characteristics of the steel were varied through two idealised stress-strain diagrams (Figure 2): elastic–perfectly plastic (analytical and numerical) and elastic–perfectly plastic with strain hardening (numerical) [11]. Geometric nonlinearity was also analysed.

The numerical analysis was validated by comparison with the analytical solutions, which were considered both with and without the influence of the axial force. After the successful validation of the numerical models (differences smaller than 5 %), a parametric analysis of the influence of the cross-sectional area of the tie rod was carried out in the range A = 1-10 cm². The analysis conducted focuses on the response of the system under horizontal loading. The term "system response" primarily denotes the load-carrying capacity of the structure and the development of plastification. Owing to pronounced convergence problems in the nonlinear numerical calculation, the emphasis was not placed on the calculation of ductility. The research was carried out under the assumption of ideal global and local stability of the system elements. Geometrical and material imperfections were not modelled. So, GMNA structural analysis was performed. Two-dimensional (plane frame) analysis without out-of-plane effects and with perfectly rigid beam-to-column connections was conducted.

2. ANALYTICAL ANALYSIS

The analytical calculation is based on the incremental "step-by-step" method. The number of steps depends on the static indeterminacy of the steel frames. In each step, a horizontal force at which a plastic hinge occurs is determined. The calculated actions are summed until the formation of a new plastic hinge leads to a collapse mechanism of the structure.

2.1. ANALYSIS WITHOUT THE INFLUENCE OF THE AXIAL FORCE

2.1.1. Unbraced steel frame

The initial phase of the research comprises the analysis of the moment capacity of steel frames, neglecting the influence of the axial force on the ultimate capacity and on the process of plastic hinge formation. The moment at which the extreme fibres of the cross-section reach the yield stress (σ_y) is defined as the yield moment (Mt = 262.2 kNm). In I-cross-sections, after the yield stress is reached in the outermost fibre, the small thickness of the flanges leads to their rapid and almost complete plastification. Because of such early development of plastification in the flanges, the I-section possesses a relatively small reserve of capacity, which may be approximated by a shape factor f ≈ 1.15. The ultimate bending moment for the considered cross-section is M* = 277.32 kNm.

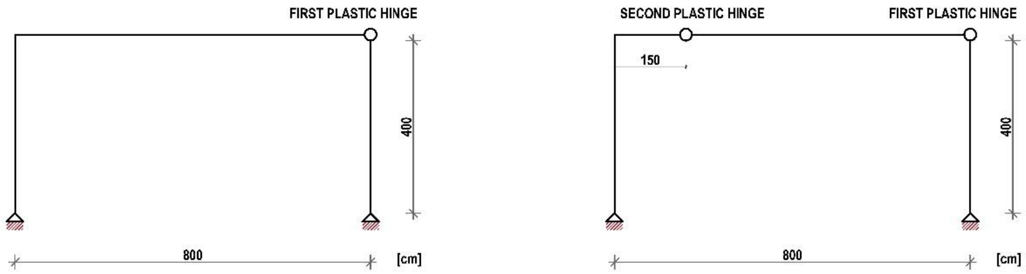


Figure 1: Development of the first(left) and second(right) plastic hinges

The first plastic hinge in the unbraced steel frame forms at the right beam-column joint. On the basis of the bending moment diagram of the considered frame and the value of the ultimate moment of the cross-section, the value of the horizontal force at which the first plastic hinge forms was determined (X_1 = 99.235 kN). After plastification of the cross-section at the right beam-column joint, a new static system is formed – a three-hinged arch (Figure 3 - left). The three-hinged arch is a statically determinate structure, and the formation of a new plastic hinge leads to collapse of the structure. The second plastic hinge forms in the lower zone of the frame beam, at a distance of 1.5 m from the left column (X_2 = 41.388 kN) – Figure 3 (right). The limit value of the horizontal force at which the collapse mechanism of the structure occurs is equal to the sum of the two previously calculated values and amounts to: $X = X_1 + X_2 = 140.573$ kN.

2.1.2. Concentrically braced steel frame

In the concentrically braced steel frame, under the action of the load, plastification of the bracing member occurs first. Since the bracing member transfers exclusively axial force, its complete plastification occurs once the yield stress is reached ($X_1 = 107.31$ kN). Through plastification, the member loses the ability to contribute to carrying the internal forces within the initial static configuration, whereby a new static system is formed in the shape of a two-hinged steel frame. The position of the first and the second plastic hinge in the further analysis remains unchanged with respect to the previously analysed static system of the unbraced steel frame – Figure 3 ($X_2 = 75.93$ kN, $X_3 = 41.02$ kN). The sum of the contributions of all plastification phases represents the ultimate load-carrying capacity of the structure: $X = X_1 + X_2 + X_3 = 224.24$ kN.

2.1.3. Analysis of the obtained results

The unbraced steel frame reaches the ultimate collapse state at a horizontal force value of 140.573 kN. The concentrically braced steel frame, owing to the engagement of the bracing member which significantly increases the stiffness and the load-carrying capacity, reaches the ultimate state only at 224.24 kN. This increase in capacity confirms that bracing increases the resistance of the structure to horizontal actions.

2.2. ANALYSIS WITH THE INFLUENCE OF THE AXIAL FORCE

Under real loading conditions, a structure is rarely subjected to bending moment alone. The presence of the axial force affects the moment capacity, so that a realistic assessment of the load-carrying capacity of the structure requires their combined action to be analysed. For the analysis of the interaction of the axial force and the bending moment, an interaction diagram (N-M) was formed, which defines the domain of admissible action of combined stresses (Figure 4).

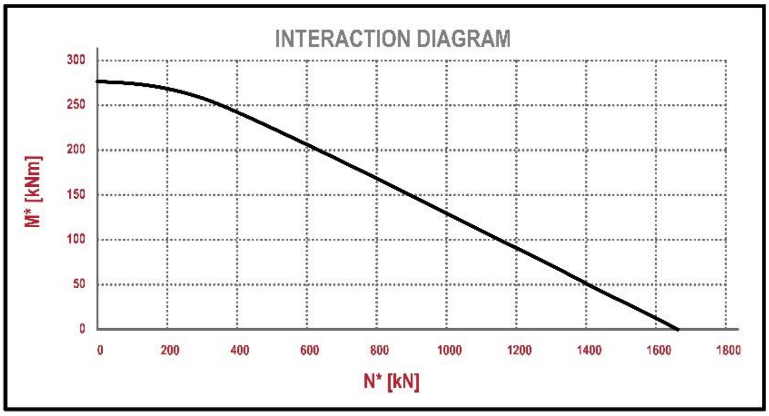


Figure 2: Moment–axial force (M–N) interaction diagram for the welded cross-section considered in this study.

After the interaction diagram had been created, the calculation was repeated with explicit consideration of the action of the axial force for both static systems. For the unbraced frame, the first plastic hinge was formed at $M_{nm} = 274$ kNm and $N_{mn} = 129.62$ kN, while the second one was formed at $M_{nm} = 276$ kNm and $N_{mn} = 68$ kN. For the concentrically braced frame, the corresponding values were M_{nm} = 271.50 kNm, N_{mn} = 168.65 kN for the first plastic hinge, and M_{nm} = 272.63 kNm, N_{mn} = 150 kN for the second one.

The results are presented in Table 1. On the basis of the presented results, it may be concluded that the axial force has a relatively small influence on the load-carrying capacity. The values of the ultimate horizontal forces obtained with the influence of the axial force expectedly decrease, while the difference with respect to the results without this influence does not exceed 5 % (Table1). With an increase in the cross-sectional area of the tie rod, the influence of the axial force on the system response is expected to become more significant. The axial force does not affect the position of plastic hinge formation in the analysed frames.

Table 1: The effect of the axial force on the system response

UNBRACED STEEL FRAME		
LEVEL OF PLASTIFICATION	HORIZONTAL FORCE CONCIDERING THE EFFECT OF AXIAL FORCE [kN]	HORIZONTAL FORCE WITHOUT CONCIDERING THE EFFECT OF AXIAL FORCE [kN]
FIRST PLASTIC HINGE	97.5	99.23
SECOND PLASTIC HINGE	136	140.57
CONCENTRIC BRACED STEEL FRAME		
LEVEL OF PLASTIFICATION	HORIZONTAL FORCE CONCIDERING THE EFFECT OF AXIAL FORCE [kN]	HORIZONTAL FORCE WITHOUT CONCIDERING THE EFFECT OF AXIAL FORCE [kN]
BRACE YIELDING	107.3	107.3
FIRST PLASTIC HINGE	180	183.24
SECOND PLASTIC HINGE	220.3	224.24

3. NUMERICAL ANALYSIS

The nonlinear numerical analysis was carried out by the finite element method in the software package *Abaqus* [10]. A nonlinear Newton's solution method was employed. The steel frame was modelled with quadratic two-dimensional B22 beam elements, while the bracing member was modelled with a T2D2 truss element. For all models, fixed incrementation was adopted with a maximum number of 1000 increments and an increment size of 0.001. The analysis was carried out in two steps. In the first step of the analysis, a vertical uniformly distributed load of 20 kN/m was applied to the structure, while in the second step a horizontal force was applied which increases incrementally up to collapse. The mesh density was varied (10 mm, 25 mm, 50 mm, 100 mm, 250 mm) and a value of 50 mm was adopted as the optimal finite element mesh size [1].

The influence of material and geometric nonlinearity on the ultimate load-carrying capacity of the structure was considered through the application of two stress-strain diagrams of steel (Figure 2).

3.1. UNBRACED STEEL FRAME

For the unbraced steel frame, the calculation was first carried out for material nonlinearity, after which geometric nonlinearity was also included in the calculation. The given diagrams (Figures 5 and 6) present the force-displacement curves for the four analysed models. Horizontal displacement of the left beam-column joint was monitored.

In the first two diagrams (Figure 5), the influence of geometric nonlinearity on the system response was analysed. Both diagrams show a consistent pattern of structural behaviour regardless of the form of the adopted material model. The force–displacement relationship is linear up to the onset of plastification. With the formation of the first plastic hinge, a drop in the stiffness of the system occurs, which is seen in the diagram as the first break, while the second break denotes the formation of the second plastic hinge. The influence of geometric nonlinearity on the system response is insignificant, with the load-carrying capacity reduced by approximately 5% for both analysed models.

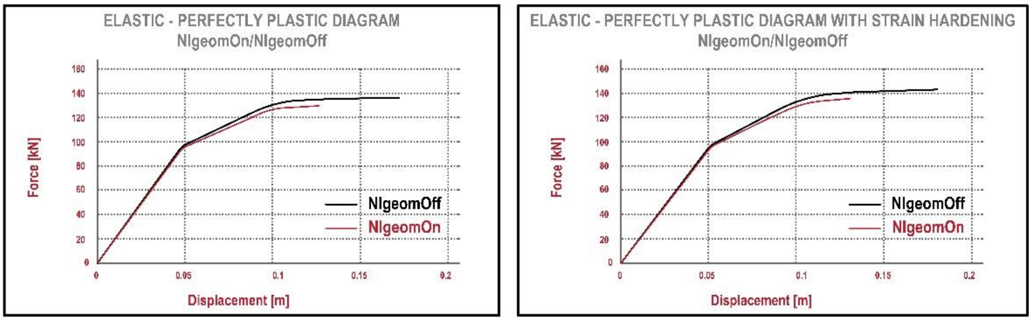


Figure 5: Results of the Influence of Geometric Nonlinearity on an Unbraced Steel Frame

In the second two diagrams (Figure 6), the influence of the strain-hardening material model on the system response was analysed. The differences between the analysed models become visible only after the formation of the first plastic hinge, that is, once the system enters the nonlinear range of behaviour. However, the observed differences are not significant. With the introduction of the strain-hardening material model, a slight increase (less than 5% in both analysed models) in the load-carrying capacity of the system occurs.

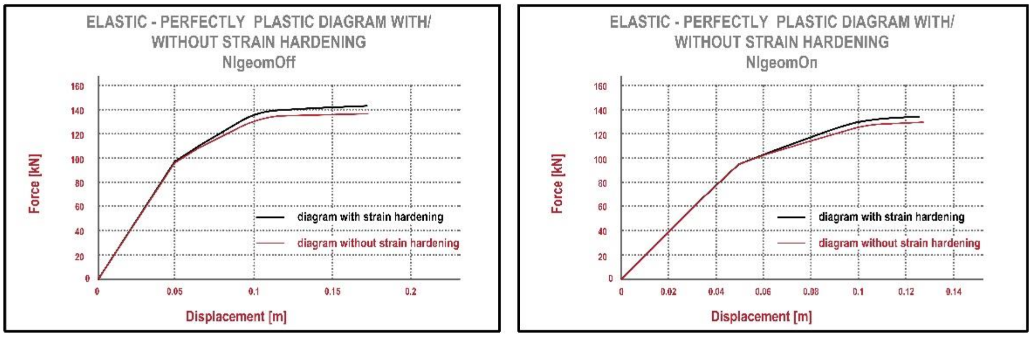


Figure 6: Results of the influence of the strain-hardening material model

The maximum stress reached in the frame amounts to 266.7 MPa, which means that the limit strain of 10 % will not be reached. Instead, the maximum strain will be achieved in the range between 1 % and 10 %. The introduction of second-order effects into the calculation does not change the qualitative pattern of behaviour of the system; their action is reflected only through a reduction in the load-carrying capacity of the analysed systems.

The position of the plastic hinges is identical in all analysed models and coincides with the results of the analytical calculation (Figure 7).

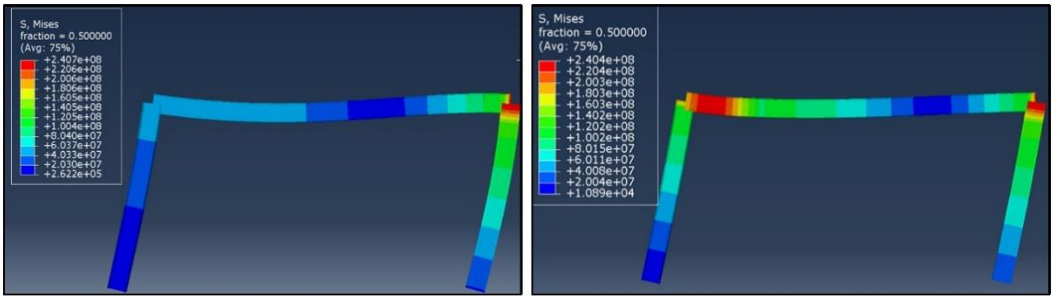


Figure 7: Formation of the First (Left) and Second (Right) Plastic Hinges:Unbraced steel frame; elastic–perfectly plastic material model; geometrically linear analysis

3.2. CONCENTRICALLY BRACED STEEL FRAME

The working methodology for the concentrically braced steel frame is entirely analogous to the procedure applied for the unbraced system. The model was formed as a single entity, whereby a cross-sectional area of 3.89 cm² was adopted for the bracing member, analogously to the analytical calculation. As in the previous system, the analysis was carried out for two stress-strain diagrams of steel (Figure 2). For the material model with strain hardening, the maximum stress amounts to 263.5 MPa. Figure 8 presents the force–displacement curves for the four analysed models. All models exhibit the same mechanical pattern of behaviour, while the differences are manifested through relatively small deviations in the ultimate forces. The models with geometric nonlinearity achieve a somewhat lower load-carrying capacity, while the introduction of hardening into the material model leads to its slight increase. The development of plastification in all analysed models is identical to that of the analytical calculation.

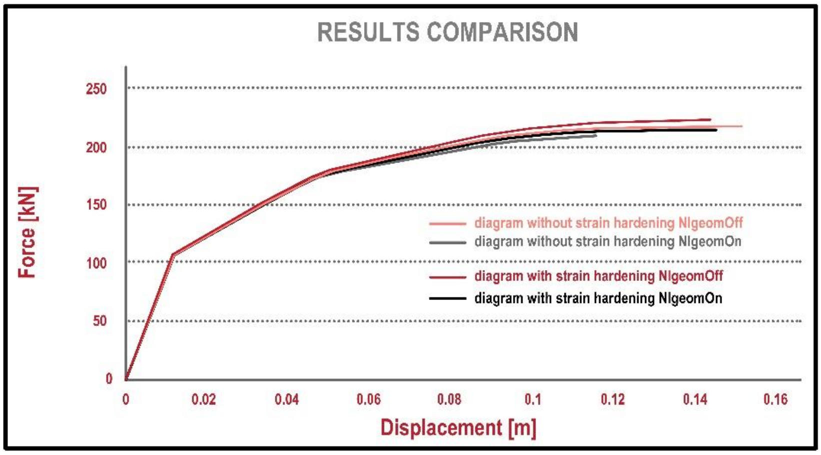
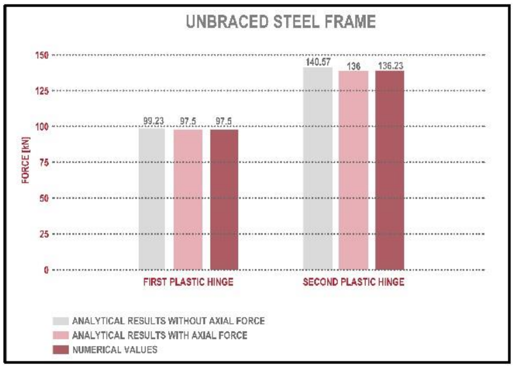


Figure 8: Results for a braced steel frame

4. VALIDATION OF THE NUMERICAL MODEL

The validation of the numerical model is based on the comparison of the analytical and numerical results (Figure 9). The analytical calculation was performed in two variants, with and without the influence of the axial force. A very good agreement between the numerical and analytical results was achieved. The differences between the obtained results are smaller than 5%. The achieved level of agreement shows that specific modelling assumptions adopted in this study provide sufficiently accurate results for the analysis of two-dimensional steel portal frames under static loading. The validated numerical model represents a reliable basis for further analysis.



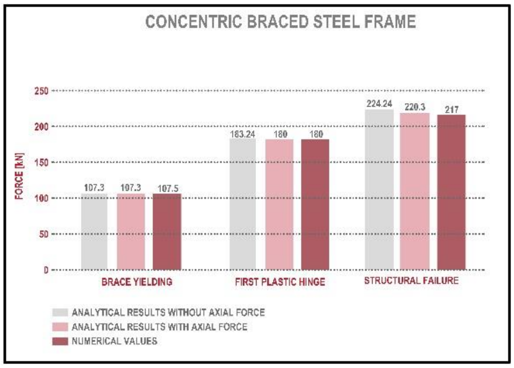


Figure 9: Validation of the numerical model

5. PARAMETRIC ANALYSIS

The examination of the influence of the tie rod area represents a key segment of the research. The research hypothesis is that the axial stiffness of the tie rod affects the stiffness and the load-carrying capacity of braced steel frames [1]. The cross-sectional area of the tie rod was varied in the range from A = 1 -10 cm² with a step of 1 cm², in order to examine the influence of its capacity on the development of internal forces, the sequence of plastic hinge formation and the final load-carrying capacity of the system. This analysis makes it possible to establish an optimal relationship between stiffness and deformability, which is of key importance for the rational design of systems subjected to horizontal force.

Figure 10 presents the results of the parametric analysis. An increase in the area of the tie rod leads to an increase in the stiffness and the load-carrying capacity of the system, whereby all curves are arranged in an almost regular sequence, from the smallest to the largest axial stiffness. In all models, the tie rod plastifies at approximately the same displacement.

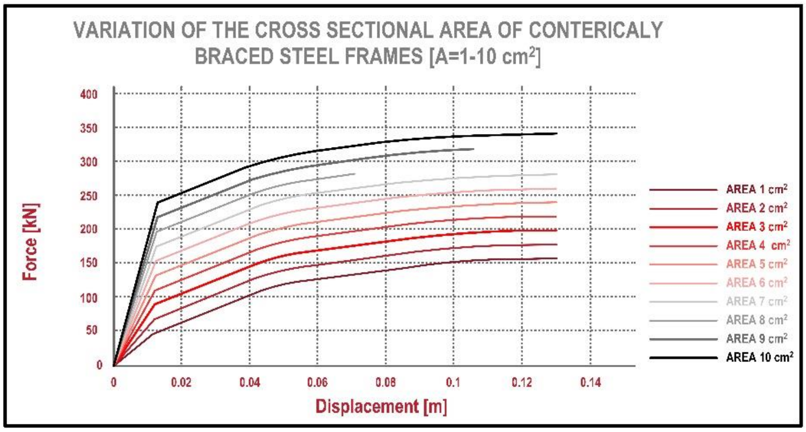


Figure 10: Effect of the tie rod cross-sectional area on the system response

For small values of the tie rod area (A = 1-2 cm²), the system exhibits a considerably lower initial stiffness (larger displacements under the same horizontal force) and the lowest load-carrying capacity.

With an increase in the area of the tie rod (A ≥ 5 cm²), the curves become steeper and the stiffness of the system increases. A stronger tie rod takes over a greater part of the horizontal load and the load-carrying capacity of the structure increases. For larger tie rod areas (A = 8-10 cm²), the system attains the greatest load-carrying capacity and stiffness. Owing to convergence problems in the numerical model, for tie rod areas of 8 and 9 cm² the calculation terminates earlier. The increase in load-carrying capacity is not linear. The greatest increase is achieved in the transition from very small tie rod areas (A = 1-3 cm²), while for larger values (A = 8-10 cm²) the increase gradually diminishes. Tie rod areas above 7 cm² provide an ever smaller contribution to the overall stiffness of the system. For smaller tie rod areas (A = 1-3 cm²), an increase in the tie rod area by 1 cm² increases the load-carrying capacity by about 10 %. However, for larger tie rod areas (A = 4-7 cm²), the additional contribution to the increase in load-carrying capacity gradually diminishes. For a tie rod area of 10 cm^2, an increase in the tie rod area by 1 cm^2, increases the load-carrying capacity of the system by 6.7 %. A change in the tie rod area in the analysed concentrically braced steel frame does not affect the position of plastic hinge formation.

Figure 11 presents the response of the unbraced and of the concentrically braced systems (A = 1 cm^2, A = 5 cm^2 and A = 10 cm^2). The unbraced steel frame is characterised by a considerably lower load-carrying capacity and stiffness. With an increase in the cross-sectional area of the tie rod, the ultimate load-carrying capacity of the concentrically braced steel frame increases. For the adopted tie rod area A = 1 cm^2, the ultimate load-carrying capacity is 13.6% greater in comparison with the basic unbraced model. With a further increase in the tie rod area to A = 5 cm^2, an increase in load-carrying capacity of 43% is achieved, while for A = 10 cm^2 the load-carrying capacity increases by approximately 60%.

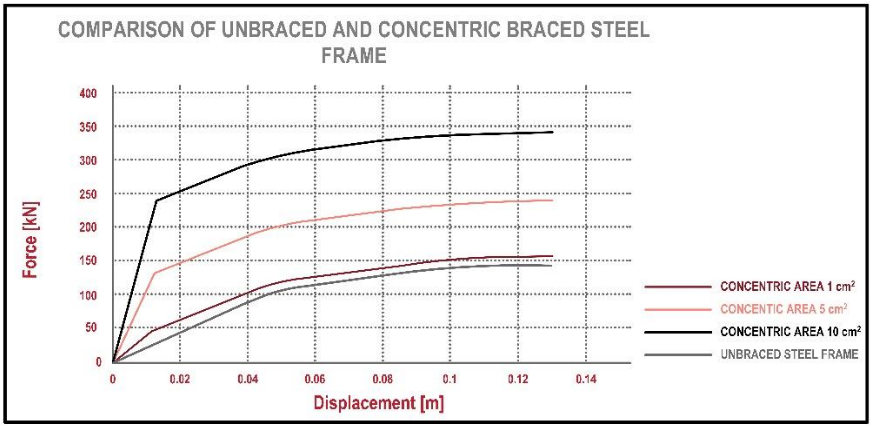


Figure 11: Comparison of two static systems

6. CONCLUDING REMARKS

Within the research, a nonlinear analysis of unbraced and concentrically braced steel frames subjected to horizontal loading was carried out. The analysis was performed by means of analytical calculation and nonlinear numerical analysis by the finite element method.

The research was performed under the following assumptions and limitations: ideal global and local stability of the system; geometrical and material imperfections were not analysed; perfectly rigid beam-to-column connections; two-dimensional (plane frame) analysis without out-of-plane effects.

The axial force has no significant influence on the formation of plastic hinges in the analysed systems. The differences in the obtained values of load-carrying capacity with and without the influence of the axial force are smaller than 5 % for the analysed tie rod area (A = 3.89 cm^2).

The introduction of the strain-hardening material model led to a slight increase in the load-carrying capacity, whereby no significant changes were observed in the development of plastification of the system.

The influence of geometric nonlinearity on the system response is negligible within the considered loading domain.

The differences between the analytical and the numerical solutions are smaller than 5%. That confirms that specific modelling assumptions adopted in this study provide sufficiently accurate results for the analysis of two-dimensional steel portal frames under static loading.

With the introduction of the tie rod, the load-carrying capacity of the system increases, whereby for the smallest analysed area (A = 1 cm²) an increase of 13.6% is achieved, while the largest analysed area (A = 10 cm²) results in an increase in load-carrying capacity of approximately 60%. The obtained results indicate that an increase in the stiffness and capacity of the tie rod significantly affects the overall response of the system, whereby the effect is more pronounced for larger values of the cross-sectional area. A change in the tie rod area does not affect the position or the sequence of plastic hinge formation in the considered systems.

On the basis of the analysis carried out, it may be concluded that the application of bracing significantly improves the resistance of steel frames to horizontal actions, while the optimal choice of tie rod stiffness represents an important factor for the efficient and rational design of steel frames.

REFERENCES

[1] Veličković, I. (2026). Master's thesis. Podgorica: University of Montenegro, Faculty of Civil Engineering.

[2] Chen, W. F., Sohal, I. Plastic Design and Second-Order Analysis of Steel Frames, Springer-Verlag, 1995.

[3] Wong, M. B. Plastic Analysis and Design of Steel Structures, Butterworth-Heinemann, 2009.

[4]Ziemian, R. D. (Ed.). Guide to Stability Design Criteria for Metal Structures, 6th ed., John Wiley & Sons, 2010.

[5] Institute for Standardization of Montenegro. MEST EN 1998-1: Eurocode 8: Design of structures for earthquake resistance – Part 1: General rules, seismic actions and rules for buildings, 2015.

[6] Santosh D., Shivamkumar, Sumit Singh, Samrat Ghatak, Nirmal Biswas, Studies on Seismic Performance of Braced and Unbraced Steel Frame by Non Linear Static Analysis, International Journal of Innovative Research in Science, Engineering and Technology, 7(5), 2018.

[7] Buđevac D. Čelične konstrukcije u zgradarstvu [Steel Structures in Building Construction], Medifarm and Građevinska knjiga, Belgrade, 1992.

[8] Dwivedi P., Deb S., Rai A., Shrivastava V. Seismic Behavior of Steel Frame Structure with and Without Bracing, International Research Journal of Engineering and Technology (IRJET), 9(7), 2022, 1543–1554.

[9] Salmon C. G., Johnson J. E., Malhas F. A. Steel Structures: Design and Behavior, 5th Edition, Pearson Education, 2009.

[10] Dassault Systèmes. Abaqus 6.11: Getting Started with Abaqus—Interactive Edition. Dassault Systèmes Simulia Corp., 2011.

[11] Yun, X., Gardner, L. Stress–strain curves for hot-rolled steels, Journal of Constructional Steel Research, 133, 2017, 36–46.